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ON MAXIMUM-LIKELIHOOD DETECTION AND ESTIMATION OF REFLECTION COEFFICIENTS

by

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Discussion of this paper is invited immediately after its presentation. Written discussions will be considered for publication in GEOPHYSICS, should the paper be accepted for publication.

ABSTRACT

A procedure is developed for obtaining maximum-likelihood estimates of the reflection coefficient sequence from seismic data. The reflection coefficient sequence is modeled as an impulsive process, where the reflection locations are statistically independent and the reflection amplitudes are uncorrelated and Gaussian distributed. The wavelet and all other parameters are assumed known. The results of this procedure are demonstrated for synthetic data.

I. Introduction

The optimal smoother method for estimating reflection coefficients from seismic reflection signals, as developed by Mendel [1-4], is a minimum-variance linear estimator which does not take into account the impulsive nature of the reflection coefficient sequence. Figure 1 shows the results of using the optimal smoother on a synthetic seismogram; the circles mark the non-zero reflection coefficients and the bars depict the corresponding estimates. As one can see, the minimum-variance estimates are generally non-zero at every time point and tend to undershoot the non-zero values of the reflection sequence. Alternatively, the (unconditional) maximum-likelihood estimator described in this paper uses the complete statistical description of the model and produces estimates with the same statistical characteristics as the process being estimated.

As with the minimum-variance estimator, we model the seismic signal as a linear convolution of the source wavelet with a reflection process, corrupted by various recording effects. These recording effects include the geophone response, aliasing filter, instrument noise, cultural noise, and can include modeling errors. Further, the signal to be estimated, namely the plane wave reflection coefficient sequence, is not the same as the signal to be convolved with the wavelet, namely the impulse response of the Earth. The difference between these two signals can be regarded as geological effects, which include spherical divergence, absorbtion, and multiples. Some of these effects can be included in our model, which is shown in Figure 2. (Including multiples would require a non-linear model of very high order, and is generally not feasible at this point.) Given this model in state vector form, and the variance of $\mu(k)$, one can perform optimal smoothing.

For our maximum-likelihood estimator, we model the Earth as a series of distinct reflectors, so that the reflection coefficient sequence will consist of a few isolated impulses. To avoid a lengthy discussion for conversion from continuous to discrete time, let us assume that these reflections can only arrive at discrete times. Due to the difficulty in modeling multiple reflections and transmission effects, let us simply ignore the additional information available from these effects. Finally, we assume that the locations (in time) of the reflections are statistically independent of each other, and that the magnitudes of these reflections are uncorrelated and Gaussian distributed.

Figure 3 depicts a synthetic reflection coefficient sequence generated using this model, and Figure 4 depicts the corresponding seismogram. For this example we have ignored all geological effects and all recording effects except for corruption by additive white Gaussian noise. This is not a constraint on the method, but was done simply to avoid complications in the presentation. Figure 4 depicts the signal from which the estimates in Figure 1 were obtained.

II. The Product Model

To simplify the estimation problem, we have found it convenient to model the reflection coefficient sequence, $\mu(k)$, as a product of two random processes of the form

$$\mu(k) = r(k)\ q(k) \tag{1}$$

where $r(k)$ is a white Gaussian process and where $q(k)$ is a binary process, which can only take on the values 0 or 1. When a reflection arrives at time k we set $q(k)=1$, and when no reflection arrives at time k we set $q(k)=0$. Therefore the process $q(k)$ is determined entirely by the locations of the reflections, so that the magnitude information must be contained in the process $r(k)$.

The statistical distribution of the $r(k)$ is completely described by its variance, C, since its mean is zero. The distribution of the $q(k)$ is given by

$$\Pr\{q(k)\} = \begin{cases} \lambda & \text{when } q(k)=1 \\ 1-\lambda & \text{when } q(k)=0 \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

where the parameter λ can be thought of as the average number of reflections per sample. Using the product model, we intend to find maximum-likelihood estimates for both the $q(k)$ and the $r(k)$, then use the invariance property of maximum-likelihood estimates to obtain estimates of the $\mu(k)$ using

$$\hat{\mu}(k) = \hat{r}(k)\ \hat{q}(k) \quad . \qquad (3)$$

III. Estimating the Reflection Amplitudes

For the case when the reflection locations are known, so that the $q(k)$ are given, we can obtain maximum-likelihood estimates of the $r(k)$ using an optimal smoother by incorporating the $q(k)$ into the state vector model. This results from $r(k)$ being a purely Gaussian process, and the maximum-likelihood estimates of a Gaussian process are the same as the minimum-variance estimates. Figure 5 demonstrates how the $q(k)$ enter into the convolution model. Figure 6 shows the resulting estimates for the case when the reflection locations are known. As one can see, these estimates are far superior to those obtained without such a priori knowledge, which were depicted in Figure 1.

Another way to estimate the $\mu(k)$ when the reflection locations are known is to incorporate the $q(k)$ into the variance model. That is, instead of using the unconditional variance

$$E\{\mu^2(k)\} = C \tag{4}$$

which was used to obtain the minimum-variance estimates, we could use the time-varying conditional variance

$$E\{\mu^2(k) \mid q(k)\} = C\, q(k) \tag{5}$$

and model the $\mu(k)$ as being the input to the convolution model. This way the optimal smoother will estimate $\mu(k)$ directly.

For the case when only the maximum-likelihood estimates $\hat{q}(k)$ are known a priori, the maximum-likelihood estimates of the $r(k)$ can be obtained using an optimal smoother by <u>incorporating the $\hat{q}(k)$ into the state vector model</u>, in place of the now unknown $q(k)$. This can be shown by factoring the likelihood expression into a term which depends on the $r(k)$ and a term which depends of the $q(k)$. The term which depends of the $r(k)$ will have the form of a conditional Gaussian density function, but where $\hat{q}(k)$ now appears in place of $q(k)$. Therefore the problem of estimating the reflection amplitudes has a known closed form solution, but it requires first estimating the reflection locations.

IV. <u>Estimating the Reflection Locations</u>

In practice the reflection locations are not known a priori and must be estimated by maximizing the likelihood expression, which is given by

$$L\{\underline{q}^{(M)} \mid \underline{z}^{(M)}\} = p\{\underline{z}^{(M)} \mid \underline{q}^{(M)}\} \Pr\{\underline{q}^{(M)}\} \tag{6}$$

using the notation

$$\underline{z}^{(M)} = [z(1), z(2), \ldots, z(M)]' \tag{7}$$

and

$$\underline{q}^{(M)} = [q(0), q(1), \ldots, q(M-1)]' \;. \tag{8}$$

Given the q(k), we can express the z(k) as linear combinations of the r(k) plus the recording noise, both of which are modeled as Gaussian; hence, the probability density function $p\{\underline{z}^{(M)} | \underline{q}^{(M)}\}$ is simply a multivariate Gaussian density function. Since the reflection locations are statistically independent, the probability distribution of the q(k) is given by

$$\Pr\{\underline{q}^{(M)}\} = \prod_{k=0}^{M-1} \Pr\{q(k)\} \tag{9}$$

where Pr{q(k)} was given by equation (2).

We can compute the likelihood for any given set of observations $\underline{z}^{(M)}$ and estimates $\hat{\underline{q}}^{(M)}$ efficiently using a Kalman filter by incorporating the estimates $\hat{q}(k)$ into the model, since the likelihood is a simple function of the resulting innovations process and variance [5]. But while we can compute the likelihood easily, <u>it is impossible to maximize it with respect to the unknown vector $\underline{q}^{(M)}$ in any reasonable amount of time</u>.

Since there are non-linear interactions between the q(k) in the likelihood expression, we cannot decouple the maximization with respect to the individual q(k), but rather must maximize with respect to the entire sequence. Since the q(k) are discrete valued, maximization will consist of comparing the likelihoods of different sequences (rather than setting a gradient to zero). Finally, since each q(k) can take on two possible values and since there are M samples in the sequence, there is a total of 2^M possible sequences $\underline{q}^{(M)}$ for which one must compare the likelihoods. When M is on the order of several hundred, this would require several centuries of computer time.

V. Suboptimal Detection of Reflections

While we may not be able to determine the global maximum of the likelihood expression, we can always design a method for detecting significant reflections; that is, we can determine relatively high likelihood or near maximum likelihood estimates for the q(k). Further, since we can easily compute the likelihood for a given estimated sequence $\hat{\underline{q}}^{(M)}$, we can compare the relative performance of different detectors. We have found one such detector which performs very well indeed.

This detector uses an approximation to the likelihood ratio for $q(k)$ of the form

$$\Lambda(k) = \frac{L\{ \hat{\underline{q}}^{(k+\ell)}(k) \ni q(k)=1 \mid \underline{z}^{(k+\ell)} \}}{L\{ \hat{\underline{q}}^{(k+\ell)}(k) \ni q(k)=0 \mid \underline{z}^{(k+\ell)} \}} \tag{10}$$

where $L\{\cdot|\cdot\}$ is given by equation (6) and where the $k+\ell$ length vector $\hat{\underline{q}}^{(k+\ell)}(k)$ is defined as

$$\hat{\underline{q}}^{(k+\ell)}(k) = [\hat{q}(0), \hat{q}(1), \ldots, \hat{q}(k-1), q(k), \lambda, \ldots, \lambda]' \quad . \tag{11}$$

This ratio compares the likelihood for two <u>particular</u> sequences which differ only in their values for $q(k)$. Further, these likelihoods use only $k+\ell$ observations, where ℓ is some small integer, rather than using all M available observations. These sequences are generated by recursively detecting $\hat{q}(0)$, $\hat{q}(1)$, ... $\hat{q}(M)$ using

$$\hat{q}(k) = \begin{cases} 1 & \text{when } \Lambda(k)>1 \\ 0 & \text{when } \Lambda(k)<1 \end{cases} \tag{12}$$

for $k=0,1\ldots M-1$. The value of $\hat{q}(j)$ in $\hat{\underline{q}}^{(k+\ell)}(k)$ for $j>k$ is replaced by the expected value,

$$E\{q(j)\} = \lambda \quad . \tag{13}$$

Figure 7 depicts the estimates which result from this detector using ℓ=5. The reflection amplitudes were estimated by incorporating the detected $\hat{q}(k)$ into the state vector model. These estimates are obviously not as good as those in Figure 6, in spite of the fact that their likelihood is many times higher. The reason for this is that the seven missed reflections were not large enough to be likely. Further, we have determined that this estimated sequence $\hat{\underline{q}}^{(M)}$ is more likely than any of its nearest neighbors.

Because of common terms in the two sequences used in equation (10), this approximate likelihood ratio can be computed by running two Kalman filters for only ℓ samples each, so that detecting the entire sequence $\hat{\underline{q}}^{(M)}$ requires about 2ℓ times as much computer time as computing its likelihood. While increasing ℓ increases the computational burden, it also improves the resulting likelihood. But since the marginal improvement from increasing ℓ drops off rapidly as ℓ increases, a relatively small ℓ will generally suffice. Figure 8 demonstrates the effect of varying ℓ.

As shown in Figure 9, as the signal-to-noise ratio decreases, the number of missed detections increases.

VI. Future Work

If one is more interested in reducing the number of missed detections than in maintaining the invariance property, then one might prefer to obtain Bayesian estimates of the reflection locations. The solution to the Bayesian detection problem consists of comparing the likelihood ratio to some threshold other than unity [6]. Since our suboptimal detector computes an approximate likelihood ratio, one can easily extend this approach to obtain approximate Bayesian estimates.

One advantage of the maximum-likelihood approach is that it can handle the case when the source wavelet, or any other model parameter, is unknown. This is done by obtaining maximum-likelihood estimates for these unknown parameters, which is a conventional non-linear optimization problem. We have shown that this approach can resolve the phase ambiguity problem in wavelet extraction. This is the area of our present interest.

Also this approach can be extended to include any bimodal or multimodal Gaussian process. For example, the Earth model may include small reflectors within the layers, so that the reflection coefficient sequence is actually drawn from two distributions: one for the layer interfaces and one for the intra-layer structures. If we model these magnitude distributions as being Gaussian with differing variances, then we can represent the reflection coefficient sequence using our product model by letting $q(k)$ take on two non-zero values.

VII. Conclusion

By representing the reflection coefficient sequence as a product of a Gaussian process and a binary process, the estimation problem is separated into two simpler problems. First, the significant reflections are detected. Second, the amplitudes are estimated using an optimal smoother which incorporates the solution to the first problem in the state vector model.

While it is impossible to definitely determine the reflection locations which maximize the likelihood in a reasonable amount of time, we have presented an efficient method for recursively detecting the significant reflections using an approximation to the likelihood ratio. This suboptimal detector has performed very well on synthetic data.

ACKNOWLEDGEMENT

The work reported on in this paper was performed at the University of Southern California, Los Angeles, California, under National Science Foundation Grant NSF ENG 74-02297 A01, Air Force Office of Scientific Research Grant AFOSR 75-2797, Chevron Oil Field Research Company Contract-76, and U. S. Geological Survey, Department of the Interior, under USGS Grant No. 14-08-0001-G-553.

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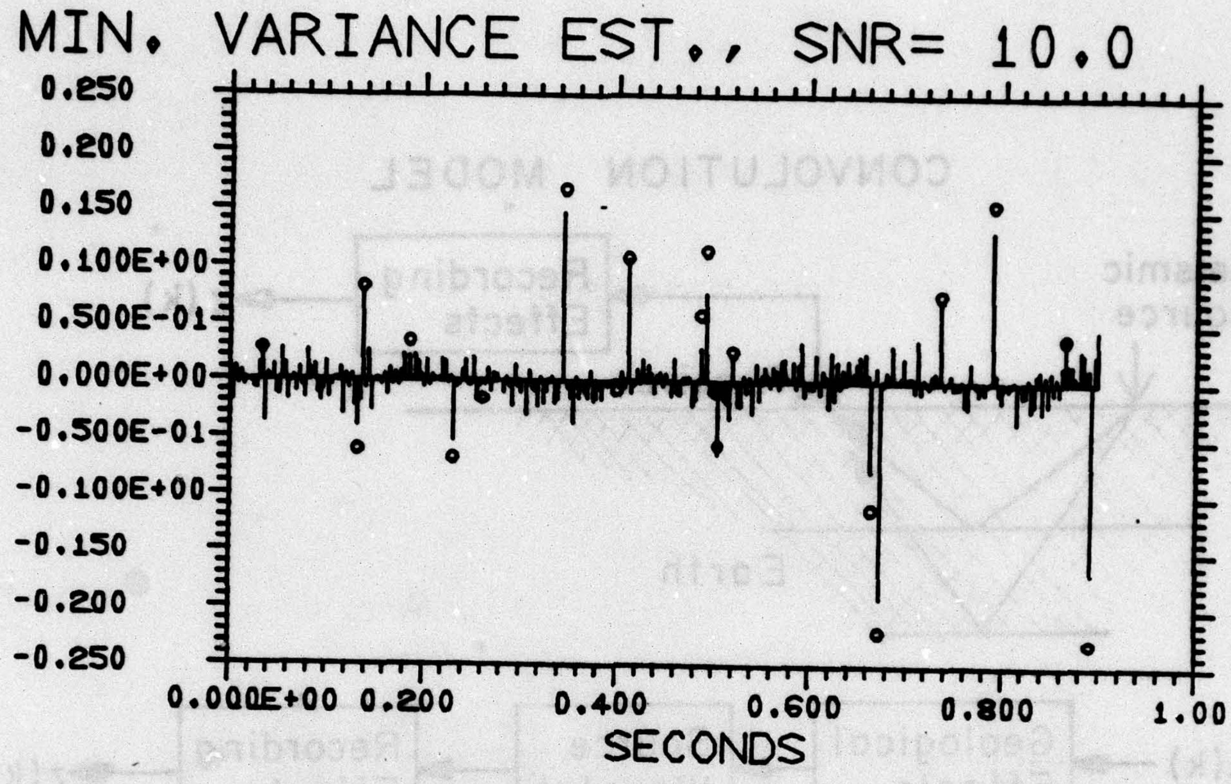
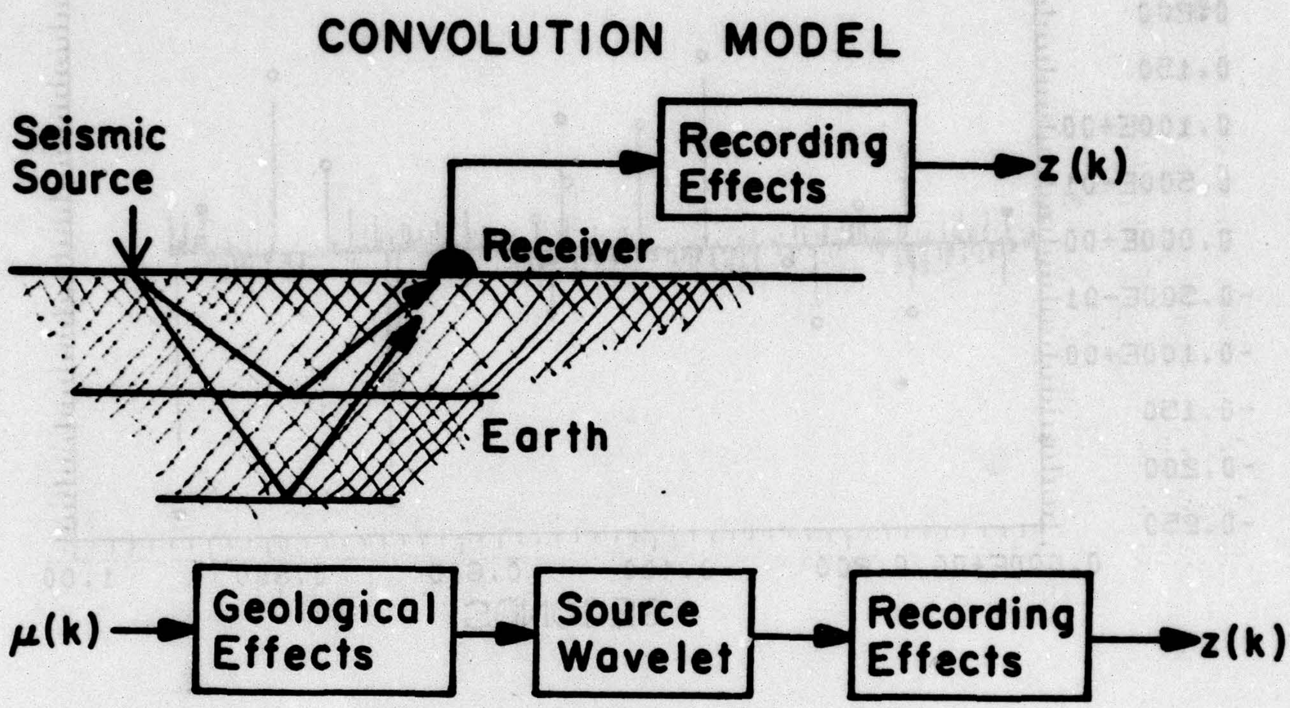


Fig. 1. Minimum Variance Estimates, SNR=10.0



$\mu(k)$ is the desired reflection sequence.

$z(k)$ is the observed data.

Fig. 2. Convolution Model

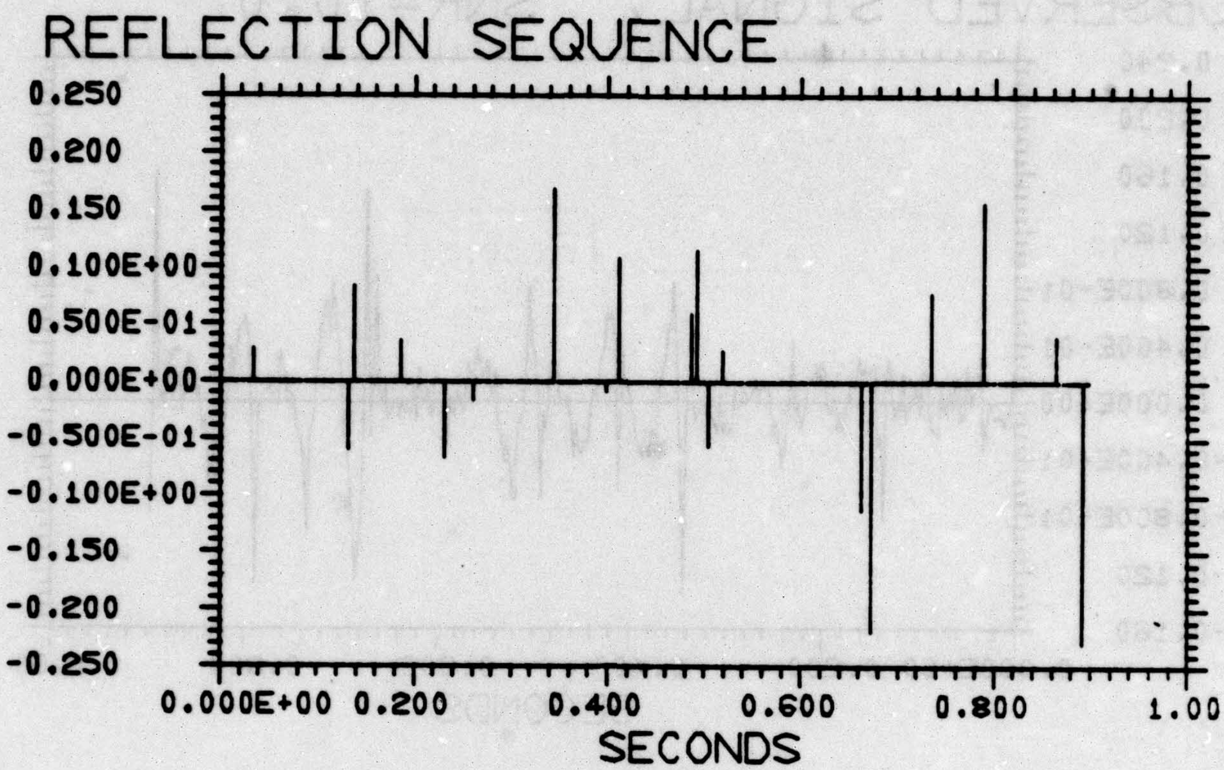


Fig. 3. Synthetic Reflection Coefficient Sequence

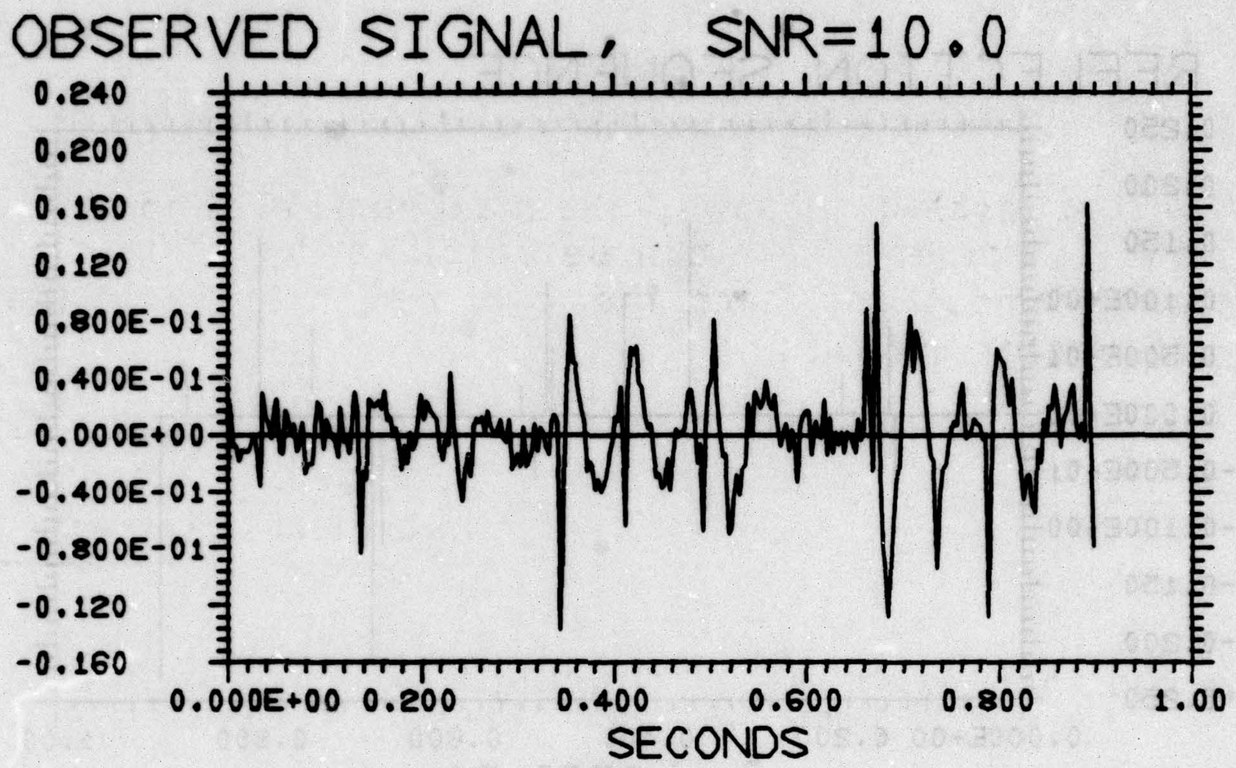
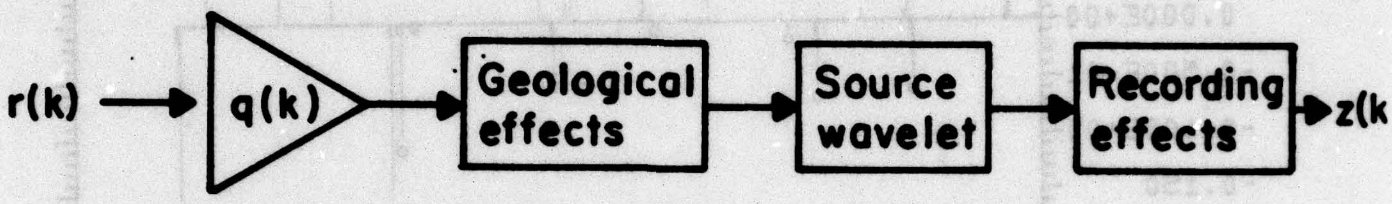


Fig. 4. Synthetic Seismogram, SNR=10.0

ESTIMATING AMPLITUDES GIVEN LOCATIONS

When the $q(k)$ are known, the maximum-likelihood estimates for the $r(k)$ can be obtained by using an optimal smoother where the $q(k)$ are included in the state vector model:



We can then estimate $\mu(k)$ using

$$\hat{\mu}(k) = \hat{r}(k)\ q(k)$$

Fig. 5. Incorporating q(k) into the Convolution Model

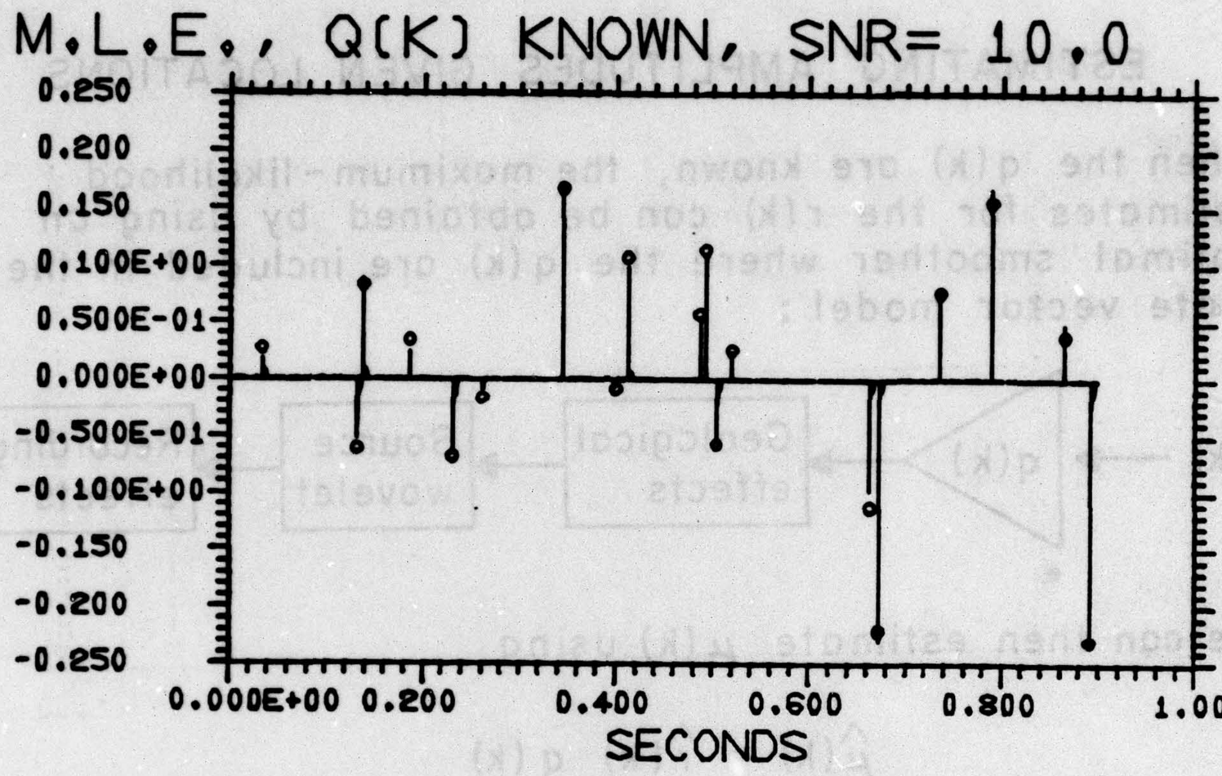


Fig. 6. Maximum Likelihood Estimates, q(k) Known

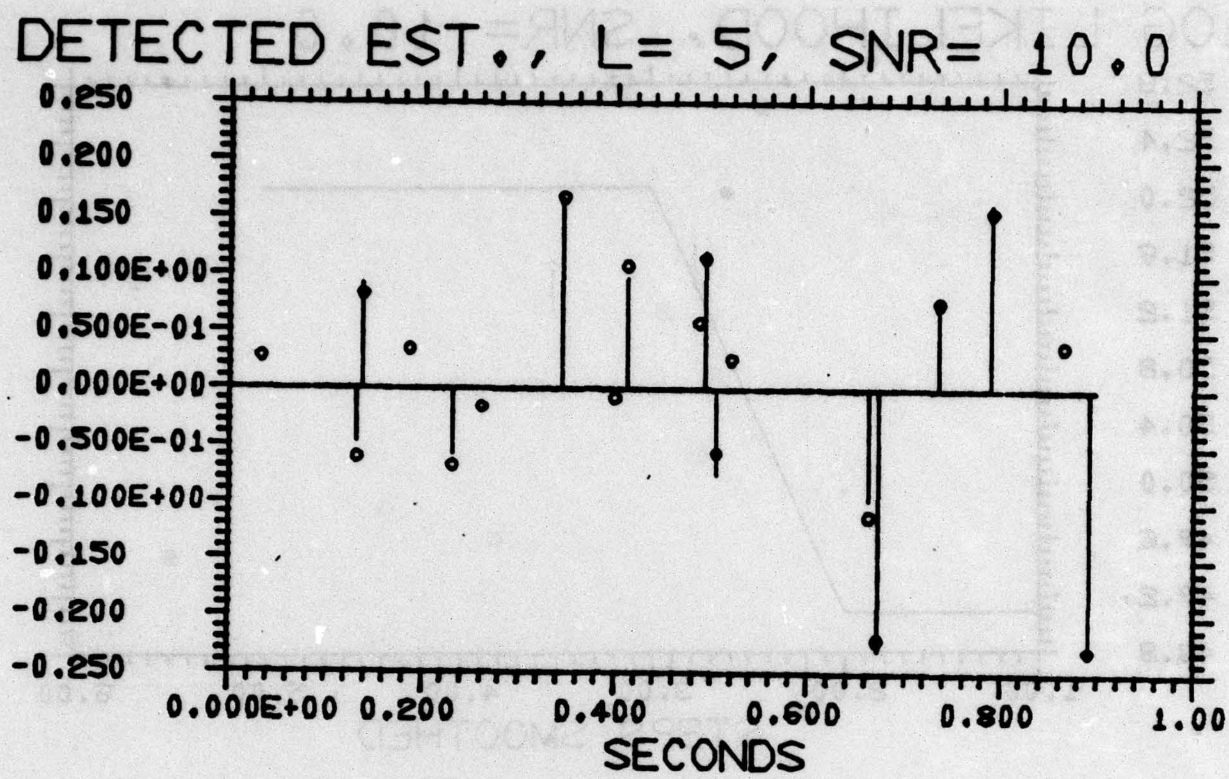


Fig. 7. Detected Estimates, SNR=10.0

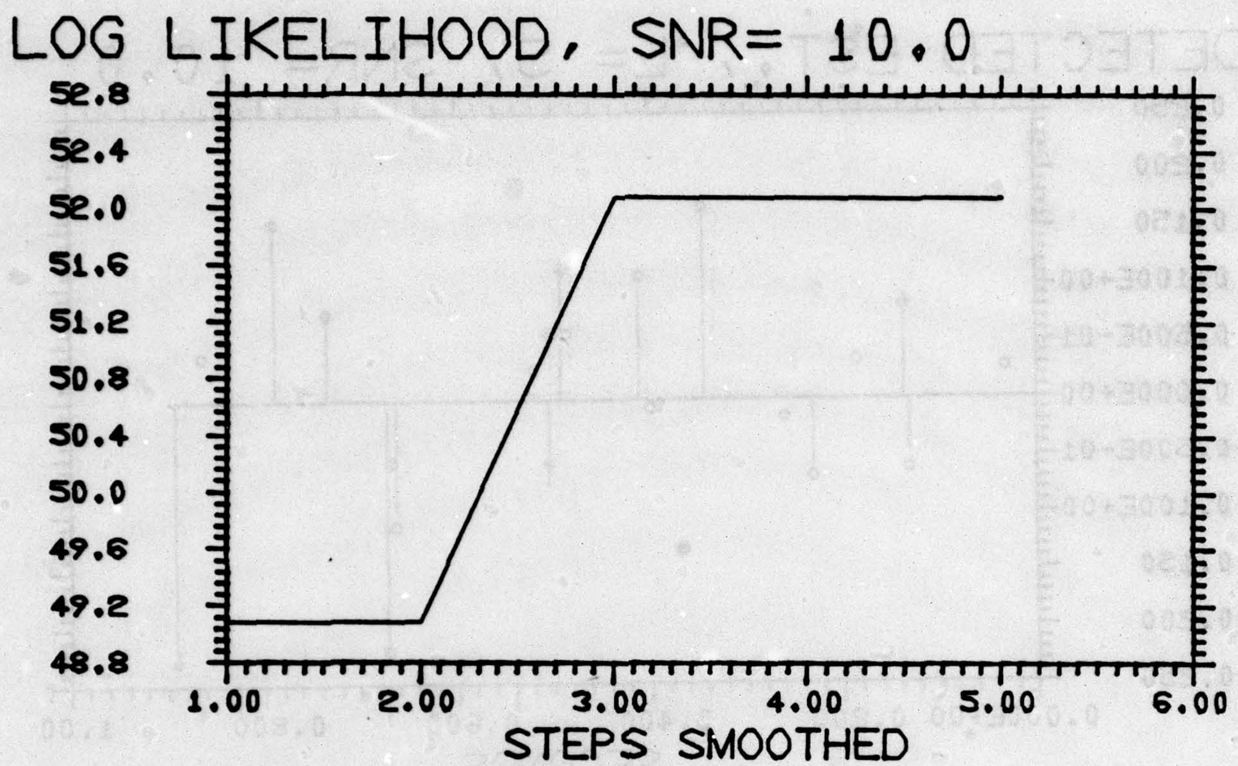


Fig. 8. Log Likelihood vs. ℓ

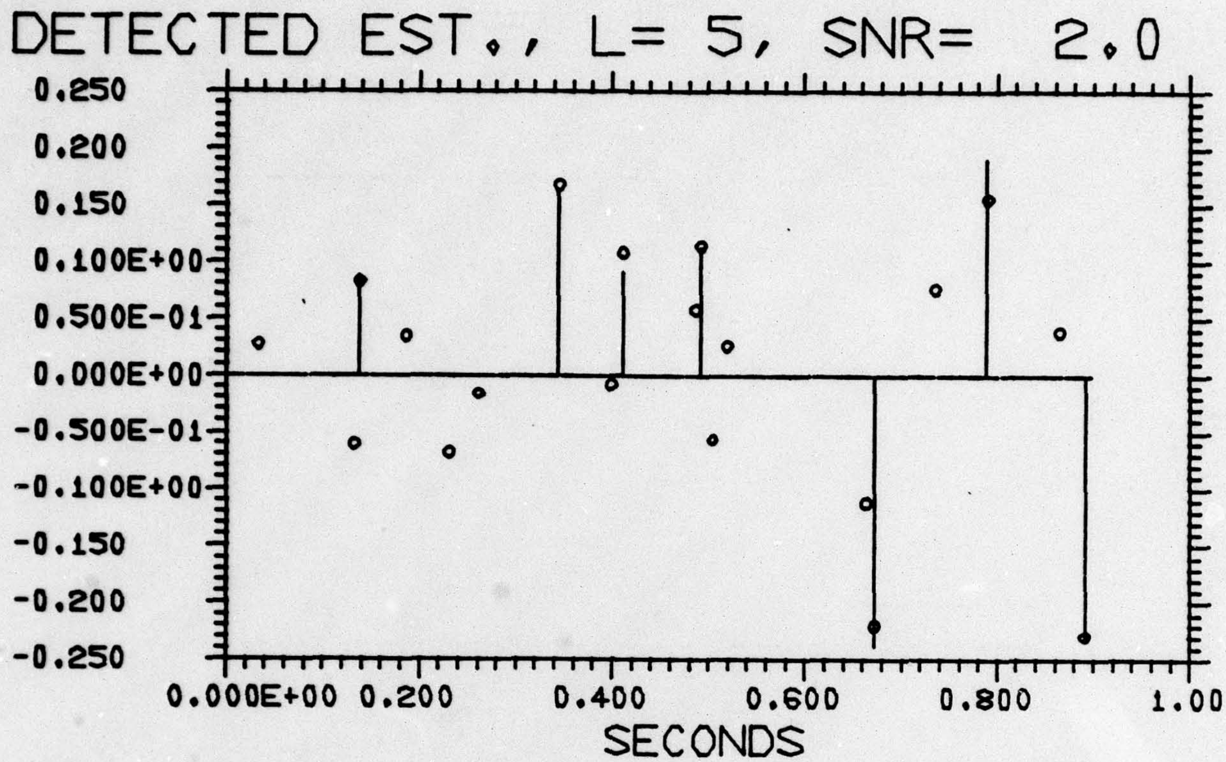


Fig. 9. Detected Estimates, SNR=2.0

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4. TITLE (and Subtitle) 6 ON MAXIMUM-LIKELIHOOD DETECTION AND ESTIMATION OF REFLECTION COEFFICIENTS.		5. TYPE OF REPORT & PERIOD COVERED 9 Interim rept. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 John Kormylo ~~and~~ Jerry M. Mendel		8. CONTRACT OR GRANT NUMBER(s) 15 ✓ AFOSR-75-2797 ✓
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Southern California Department of Electrical Engineering Los Angeles, California 90007		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 16 61102F 2304 ~~A1~~ 17 A1
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332		12. REPORT DATE 11 1978 13. NUMBER OF PAGES 24
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 25p		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A procedure is developed for obtaining maximum-likelihood estimates of the reflection coefficient sequence from seismic data. The reflection coefficient sequence is modeled as an impulsive process, where the reflection amplitudes are uncorrelated and Gaussian distributed. The wavelet and all other parameters are assumed known. The results of this procedure are demonstrated for synthetic data.

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